# A Market Demand Curve Construction for a Good by Using Bivariate Probability Distribution Method

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#### **Abstract**

A demand curve is usually constructed by using utility function method. Practically, it is difficult to estimate a market demand curve for a good by means of the method. The basic assumption of the study is that a demand curve for a good is influenced by two variables; the highest price a consumer is willing to pay for the good and the highest price a consumer is able to afford for the good. Then through market surveys, one can acquire these prices the consumer is willing to pay and those he is able to afford. Therefore, a market demand curve can be constructed by applying these data.

#### 1. Introduction

Alfred Marshall (1890) had used equal marginal utility principle to form an individual demand curve. Some economists have used another approach, indifference curve analysis, to form an individual demand curve. Then a market

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demanded curve is the "horizontal sum" of each individual's demand curve. At each price the quantity demand in the market is the sum of amounts each individual demands.

The methods mentioned above are abstract, and in practice, it is difficult to estimate a market de nand curve form by using these methods. Instead of using the two methods, many scholars apply econometric methods to form a specific good demand curve in empirical research. For example, Archibald and Gillingham (1980); Kayser (2000) use statistical data to estimate the gasoline demand. Mannering and Winston (1985) use households data to estimate automobile choice and usage. The researchers above apply multiple regression equation with historical statistical data to estimate a market demand curve. Recently, Afriat (1967), Diewert (1973, 1985); Landsburg (1981); Varian (1982, 1983, 1985); Chavas and Cox (1997) use nonparametric methods to analyze consumer behavior with observed data without ad hoc specification of a function form for preferences or/and demand function.

However, these empirical studies above all need observed historical data to meet the demand for a specific good. But under some circumstances, historical data for a new product may not be observed, and it will be difficult to estimate the demand curve for a new product which has no historical data. Therefore, according to the definition of the quantity demanded of a good, which is the amount of the good that consumers are willing and able to purchase at a certain price during a specific period, we can know the quantity demanded of a good is influenced by two variables: the price a consumer is willing to pay, and the price he is able to afford. Now, we try to use these two variables and bivariate probability distribution method to construct a market demand curve. The method is presented in Section 2. Moreover, if the joint probability distribution of the prices the consumers are willing to pay and the prices the consumers are able to afford can be normalized, the market demand curve will be constructed by using bivariate normal probability distribution approach. The result is presented in Section 3. According to properties derived by the newly constructed market demand curve in Section 3, we will further point out the nature of inferior goods for the entire market in Section 4, and suggest some strategies to raise a firm's revenue in Section 5. The concluding section of the article will consist of a concise summary.

# 2. A Market Demand Curve Construction by Using Bivariate Probability Distribution Approach

In our study, we must emphasize that the market demand curve will not be constructed by traditional utility method, but a new method. Therefore the properties of utility function will not be discussed in this paper.

First, regardless of the consumer's income, there is a maximum subjective value for any specific good in a consumer's mind. For example, in a consumer's mind, a BENZ car may be worth \$500,000. Therefore, if the price of the BENZ car is \$500,000 or below, he may be willing to purchase the BENZ car, and we call the price of \$500,000 the willing price of the consumer for the BENZ car. But whether

the consumer really buys the car depends not only on the willing price but also on his affordability.

Next, a consumer will need many goods in his life, he must distribute his income to these goods. Considering the necessity of a specific good and his own income, he must have a highest budget constraint for any specific good. If the price of the good is lower than the highest budget constraint, the consumer will have the ability to buy it, and we call the price the consumer can afford to pay the affordable price of the consumer for the good.

To illustrate this statement, we will assume that there are ten consumers in a certain area. Each of the consumers will be asked to list his affordable highest price, x, and his willing price, y, for a specific good. Then, by using market surveys, we can get all those combinations of the prices in Table 1.

Table 1

The combination of individual's ability to buy a good at the price x and his willingness to pay the highest price, y, for the good

Individual consumer	The highest price, x, an individual is able to afford	The highest price, y, an individual is willing to pay	
1	100	75	
2	90	70	
3	80	65	
4	80	65	
5	80	60	
6	70	70	
7	70	55	
8	60	70	
9	60	60	
10	50	60	

For individual 1, considering his income and budget, if the price of the good is \$100, he will have the ability to buy the good, but he thinks the good is worth only \$75, that is, he is willing to pay \$75 for the good in his mind; so he will buy the good only if the price is \$75 or below. On the contrary, for individual 8, considering his income allocation, he has the budget of only \$60 to buy the good. Even though he thinks the good is worth \$70, he still can't buy the good due to his budget constraint. Therefore, the price of the good must drop to \$60 or below, and then he might be able to buy it. Based on the mention above, the highest price for an individual who actually pays will be the lower one chosen from the affordable price, x, and the willing price, y. Now we call the highest price for an individual who actually pays the highest valid price.

According to the mention above, Table 2 will be derived by using the data in Table 1.

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Individual consumer	The highest valid price an individual is actually able to pay		
1	$75 = \min\{100, 75\}$		
2	$70 = \min\{90, 70\}$		
3	$65 = \min \{80, 65\}$		
4	$65 = \min \{80, 65\}$		
5	$60 = \min \{80, 60\}$		
6	$70 = \min \{70, 70\}$		
7	55 = min {70, 55}		
8	60 = min {60, 70}		
9	60 = min {60, 60}		

Table 2

The highest valid price each individual is actually able to pay for a good.

To strike a bargain, the price of a good must not be greater than the highest valid price an individual is actually able to pay. Therefore, if the price of the good ranges from 50 to 80, actual quantity demanded can be seen at Table 3.

 $50 = \min \{50, 60\}$ 

Table 3
Quantity demanded when the price is set at different level.

Price of the good	Individual consumer who will buy	Actual quantity demanded
50	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	10
55	1, 2, 3, 4, 5, 6, 7, 8, 9	9
60	1, 2, 3, 4, 5, 6, 8, 9	8
65	1, 2, 3, 4, 6	6
70	1, 2, 6	3 .
75	1	1

The data in Table 3 can be plotted to form a demand curve graphically. We have done this in Figure 1.

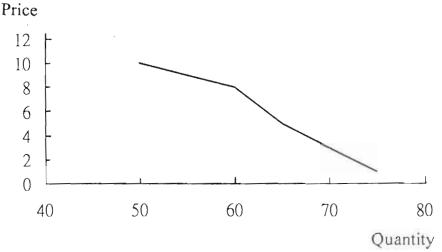


Figure 1: The market demand curve.

Based on the data above, we can construct the market demand curve by using probability distribution function. For example, the number of potential consumers for a good in an area is expressed as N during a certain period of time;  $x_i$  represents the price individual consumer i is able to afford;  $y_i$  represents the highest price he is willing to pay; f(x, y) is the bivariate probability density function, and symbol  $\Theta$  represents the expression of  $x_i$   $\Theta$   $y_i = \min\{x_i, y_i\}$ . When the price of the good is set at p, the required condition to meet individual i's valid demand can be expressed as  $x_i$   $\Theta$   $y_i$ . Therefore quantity demanded, Q, and the price, p, have the following relation:

$$Q = N . P_r \{ (x, y) | x \Theta \ y \ge p \}$$

$$= N \left[ \sum_{\substack{y = x \\ y \ge p}} \sum_{\substack{x \ge y \\ x \ge y}} f(x, y) + \sum_{\substack{x = y \\ x \ge p}} \sum_{\substack{y \ge x \\ y \ge x}} f(x, y) \right]$$
(1)

where  $P_r$  { } represents the value of probability function.

For example, if p equals \$60, quantity demanded will be 8.

$$Q = N \left[ \sum_{\substack{y = x \\ y \ge 60 \ x \ge y}} \sum_{\substack{x = y \\ x \ge 60 \ y \ge x}} f(x, y) + \sum_{\substack{x = y \\ x \ge 60 \ y \ge x}} \sum_{\substack{y \le 60 \ x \ge y}} f(x, y) \right]$$

Now, If f(x, y) is a bivariate probability function, equation (1) can be written as

$$Q = N \left[ \int_{p}^{\infty} \int_{y}^{\infty} f(x, y) dx dy + \int_{p}^{\infty} \int_{x}^{\infty} f(x, y) dy dx \right]$$
 (2)

By equation (2), it is easy to derive that

= 8.

$$\frac{\partial}{\partial p} \int_{p}^{\infty} \int_{y}^{\infty} f(x, y) dx dy < 0$$
, and  $\frac{\partial}{\partial p} \int_{p}^{\infty} \int_{x}^{\infty} f(x, y) dy dx < 0$ 

Consequently, we can get  $\frac{\partial Q}{\partial P} < 0$ .

Based on the analysis, equation (2), the market demand function still meets the law of demand: Other things being equal, when the price of a good rises, the quantity demanded of the good falls.

Furthermore, equation (2) can be simplified either as equation (3) or eqn (4).

$$Q = N \left[ \int_{p}^{\infty} \int_{p}^{\infty} f(x, y) dx dy \right]$$
 (3)

$$Q = N \left[ 1 - F_{x}(P) - F_{y}(P) + \int_{-\infty}^{p} \int_{-\infty}^{p} f(x, y) dx dy \right]$$
 (4)

where

$$F_{X}(P) = \int_{-\infty}^{p} \int_{-\infty}^{\infty} f(x, y) dy dx, F_{Y}(P) = \int_{-\infty}^{p} \int_{-\infty}^{\infty} f(x, y) dx dy.$$

If g (x) and h (y) are the marginal densities of f (x,y), respectively, and let us define  $\varepsilon_f(x,y) = f(x,y) - g(x) h(y)$ , equation (4) will become

$$Q = N \left[ 1 - F_{x}(p) - F_{y}(p) + \int_{-\infty}^{p} g(x) dx \int_{-\infty}^{p} h(y) dy \right]$$

$$+ \int_{-\infty}^{p} \int_{-\infty}^{p} \varepsilon_{f}(x, y) dx dy$$

$$= N \left[ \overline{F}_{x}(p) \right] \left[ \overline{F}_{y}(p) \right] + N \cdot \int_{-\infty}^{p} \int_{-\infty}^{p} \varepsilon_{f}(x, y) dx dy$$
(5)

where

$$\overline{F}_{x}(p) = 1 - F_{x}(p), \overline{F}_{y}(p) = 1 - F_{y}(p).$$

Under certain circumstances, variable x and y are random independent variables, f(x,y) will become g(x) h(y). Then equation (5) can be written as

$$Q = N \left[ \overline{F}_{x}(p) \right] \left[ \overline{F}_{y}(p) \right].$$
 (6)

From equation (5), we can find that an individual demand is influenced not only by his affordability and willingness but also by the correlation,  $\varepsilon_f(x, y)$ , between the variable x and y. Now, if the distribution can be normalized, then we can use bivariate normal distribution approach to construct the market demand function. Furthermore, we can infer how a market demand curve is influenced by the consumers' affordability, willingness and the correlation between affordability and willingness. The results are presented in Section 3.

# Effects of Changes in Affordability, in Willingness and in the Correlation Between Them.

In this section, we will use the bivariate normal distribution model to study how the market demand for a good is affected by changes in consumer's affordability, in willingness, and in the correlation between them.

As we pointed out in Section 2, if the joint distribution of the consumer's affordable price, x, and willing price, y, can be normalized to meet the requirement of bivariate normal distribution, we can get the following equation:

$$f(x, y) = \frac{1}{2 \pi \sigma_x \sigma_y \sqrt{1 - \rho^2}}$$

$$\times e^{\frac{-1}{2(1 - \rho^2)} \left[ \left( \frac{x - \mu_x}{\sigma_x} \right)^2 - 2 \rho \left( \frac{x - \mu_x}{\sigma_x} \right) \left( \frac{y - \mu_x}{\sigma_y} \right) + \left( \frac{y - \mu_y}{\sigma_y} \right)^2 \right]}$$
(7)

where

 $\mu_x$ : mean value of x;

 $\mu_v$ : mean value of y:

 $\sigma_x$ : standard deviation of x;

 $\sigma_{v}$ : standard deviation of y;

 $\rho$ : coefficient of correlation between variables x and y.

Then the market demand function can be expressed as

$$Q = N \int_{p}^{\infty} \int_{p}^{\infty} \frac{1}{2 \pi \sigma_{x} \sigma_{y} \sqrt{1 - \rho^{2}}}$$

$$\times e^{\frac{-1}{2(1 - \rho^{2})} \left[ \left( \frac{x - \mu_{x}}{\sigma_{x}} \right)^{2} - 2 \rho \left( \frac{x - \mu_{x}}{\sigma_{x}} \right) \left( \frac{y - \mu_{x}}{\sigma_{y}} \right) + \left( \frac{y - \mu_{y}}{\sigma_{y}} \right)^{2} \right]_{dx dy}}$$
(8)

Now, if let us define

$$u = \frac{x - \mu_x}{\sigma_x}, \ v = \frac{y - \mu_y}{\sigma_y},$$

then Q can be written as follows.

$$Q = N \int_{\frac{p - \mu_{y}}{\sigma_{y}}}^{\infty} \int_{\frac{p - \mu_{x}}{\sigma_{x}}}^{\infty} \frac{1}{2 \pi \sqrt{1 - \rho^{2}}} e^{\frac{-1}{2(1 - \rho^{2})} (u^{2} - 2\rho u v + v^{2})} du dv$$
(9)

Furthermore, suppose

$$w = \frac{u - \rho v}{\sqrt{1 - \rho^2}}$$

equation (9) can be expressed as

$$Q = N \int_{\frac{p-\mu_{y}}{\sigma_{y}}}^{\infty} \left( \frac{p-\mu_{x}}{\sigma_{x}} - \rho v \right) / \sqrt{1-\rho^{2}} \frac{1}{2\pi} e^{\frac{-1}{2}(w^{2}+v^{2})} dw dv$$
(10)

By equation (10), it is easy to derive that

$$\frac{\partial Q}{\partial \mu_x} > 0$$
 and  $\frac{\partial Q}{\partial \mu_y} > 0$ 

Consequently, we can get the following property 1 and property 2.

**Property 1**: Other things being equal, as the consumer's average income increases, the average price they are able to afford,  $\mu_x$ , will increase at the same time. Then quantity demanded will also increase.

**Property 2:** As consumers are increasing their willing price, the average price,  $\mu_v$  will increase at the same time. Then quantity demanded will also increase.

Now, taking the partial derivative of Q with respect to  $\rho$ , we can obtain  $\frac{\partial Q}{\partial \rho} > 0$  (the proof is provided later), and get the following property 3 and property 4.

**Property 3**: Other things being equal, if the consumers' income and their willingness are positively correlated,  $\rho \in (0, 1)$ , then quantity demanded will increase as  $|\rho|$  increases. The stronger the correlation  $|\rho|$  is, the larger the quantity demanded is.

**Property 4**: Other things being equal, if the consumers' income and their willingness are negatively correlated  $\rho\epsilon$  [- 1,0], then quantity demanded will decrease as  $|\rho|$  increases (i.e.  $\rho$  decreases). The stronger the correlation  $|\rho|$  is, the smaller the quantity demanded is.

### Proof of Property 3 and 4

By using equation (4) and (7), if we,

Let

$$Z = \int_{-\infty}^{p} \int_{-\infty}^{p} \frac{1}{2 \pi \sigma_{x} \sigma_{y} \sqrt{1 - \rho^{2}}}$$

$$\times e^{\frac{-1}{2 (1 - \rho^{2})} \left[ \left( \frac{x - \mu_{x}}{\sigma_{x}} \right)^{2} - 2 \rho \left( \frac{x - \mu_{x}}{\sigma_{x}} \right) \left( \frac{y - \mu_{y}}{\sigma_{y}} \right) + \left( \frac{y - \mu_{y}}{\sigma_{y}} \right)^{2} \right]_{dx dy}}$$

we can get the following equation.

$$\frac{\partial Q}{\partial \rho} = N \frac{\partial Z}{\partial \rho} \tag{11}$$

Now, let

$$u = \frac{x - \mu_x}{\sigma_x}, \ v = \frac{y - \mu_y}{\sigma_y},$$

Z can be written as follows.

$$Z = \int\limits_{-\infty}^{\frac{p-\mu_{_{\boldsymbol{y}}}}{\sigma_{_{\boldsymbol{y}}}}} \int\limits_{-\infty}^{\frac{p-\mu_{_{\boldsymbol{x}}}}{\sigma_{_{\boldsymbol{x}}}}} \frac{1}{2\,\pi\,\sqrt{1\,-\,\rho^2}} \,\,e^{\,\displaystyle\frac{-1}{2\,(1-\rho^2\,)}\left(\,u^2\,-\,2\,\rho\,u\,v\,+\,v^2\,\right)} \,du\,dv$$

$$= \int_{-\infty}^{\mathbf{p} - \mu_{y}} \left( \frac{\mathbf{p} - \mu_{x}}{\sigma_{x}} - \rho \, v \right) / \sqrt{1 - \rho^{2}}$$

$$= \int_{-\infty}^{\mathbf{p} - \mu_{y}} \frac{1}{2 \, \pi} \, e^{\frac{-1}{2} \left( \mathbf{w}^{2} + \mathbf{v}^{2} \right)} \, d\mathbf{w} \, d\mathbf{v}$$

where

$$w = \frac{u - \rho v}{\sqrt{1 - \rho^2}}.$$

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Taking the partial derivative of Z with respect to  $\rho$ , we obtain

$$\frac{\partial Z}{\partial \rho} = \int_{-\infty}^{\mathbf{p} - \mu_{\mathbf{y}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^{2}}{2} - \frac{(-p + \mu_{\mathbf{x}} + v \rho \sigma_{\mathbf{x}})^{2}}{(2 - 2\rho^{2}) \sigma_{\mathbf{x}}^{2}}}$$

$$\times \left[ \frac{v}{\sqrt{2 - 2\rho^{2}}} + \frac{2\rho \left( (-p + \mu_{\mathbf{x}} + v \rho \sigma_{\mathbf{x}}) \right)}{\sqrt{(2 - 2\rho^{2})^{3} \sigma_{\mathbf{x}}}} \right] dv$$

$$= \frac{1}{2\pi \sqrt{1 - \rho^{2}}} e^{-\frac{v^{2}}{2} - \frac{(-p + \mu_{\mathbf{x}} + v \rho \sigma_{\mathbf{x}})^{2}}{(2 - 2\rho^{2}) \sigma_{\mathbf{x}}^{2}}} \left| \frac{p - \mu_{\mathbf{y}}}{\sigma_{\mathbf{y}}} - \infty \right|$$

$$= \frac{1}{2 \pi \sqrt{1 - \rho^2}} e^{\frac{-1}{2(1 - \rho^2)} \left[ \left( \frac{p - \mu_x}{\sigma_x} \right)^2 - 2 \rho \left( \frac{p - \mu_x}{\sigma_x} \right) \left( \frac{p - \mu_y}{\sigma_y} \right) + \left( \frac{p - \mu_y}{\sigma_y} \right)^2 \right]} > 0$$
(12)

Together with equation (11) and (12), it leads

$$\frac{\partial Q}{\partial \rho} > 0.$$

Furthermore, taking the partial derivative of total revenue, TR, with respect to p, we can get the following properties (the proof is provided later).

**Property 5**: If price elasticity of demand is greater than 1, and consumers' income and their willingness are positively correlated, a firm's sales revenue will increase as the correlation  $(|\rho|)$  is increasing.

**Property 6**: If price elasticity of demand is greater than 1, and consumers' income and their willingness are negatively correlated, a firm's sales revenue will decrease as the correlation  $(|\rho|)$  is increasing (i.e.  $\rho$  is decreasing).

**Property** 7: If price elasticity of demand is less than 1, and consumer's income and their willingness are positively correlated, a firm's sales revenue will decrease when the correlation ( $|\rho|$ ) is increasing.

**Property 8**: If price elasticity of demand is less than 1, and consumers' income and their willingness are negatively correlated, a firm's sales revenue will increase when the correlation  $(|\rho|)$  is increasing (i.e  $\rho$  is decreasing).

Properties 5, 6, 7 and 8 are proven as following:

**Proof**: According to law of demand and the definition of price elasticity of demand,  $|E_d|$ , if a good's price is P (Q), then  $|E_d|$ , and total sales revenue of a firm, TR, can be expressed as follows.

$$\left| E_{d} \right| = \left| \frac{\frac{\partial Q}{Q}}{\frac{\partial P(Q)}{P(Q)}} \right| = -\frac{\frac{\partial Q}{Q}}{\frac{\partial P(Q)}{P(Q)}}$$

$$TR = P(Q) \times Q.$$

Taking the partial derivate of TR with respect to r, we obtain

$$\frac{\partial TR}{\partial \rho} = P(Q) \times \frac{\partial Q}{\partial \rho} + \frac{\partial P(Q)}{\partial \rho} \times Q$$

$$= P(Q) \times \frac{\partial Q}{\partial \rho} \times \left[ 1 + \frac{\frac{\partial P(Q)}{\partial (Q)}}{\frac{\partial Q}{\partial (Q)}} \right]$$

$$= P(Q) \times \frac{\partial Q}{\partial \rho} \times \left[ 1 - \frac{1}{|E_d|} \right].$$

By equation (13), we have got

$$\frac{\partial Q}{\partial \rho} > 0$$
,

therefore

$$\begin{cases} \frac{\partial TR}{\partial \rho} > 0, & \text{if } |E_d| > 1 \\ \\ \frac{\partial Q}{\partial \rho} < 0, & \text{if } |E_d| < 1 \end{cases}$$

## 4. The nature of inferior goods for the entire market

Traditional economic theory states that demand of a normal good will increase as a consumer's income increases. On the contrary, demand of an inferior good decreases as a consumer's income increases; however, the reason for the phenomena has seldom been discussed since before. Property 1 in this study, states that the quantity demanded in the market will increase as consumers' average income increase. Hence we find that the nature of inferior goods for the entire

market depends on negative correlation between the consumers' affordability and willingness, and the negative correlation concerning inferior goods increases (i.e.  $|\rho|$  increases and  $\rho$  decreases) as the consumers' average income increases. Even though the increase of the consumer's average income promotes his consumption, the "perverse" correlation effect will be strong enough to make total quantity demanded of an inferior good fall. For example, even though the consumers' average income is raised, the market demand for potatoes, an inferior good, will decrease.

### 5. Marketing Strategies

Based on the analysis of section 3, a firm can form its marketing strategies when the correlation between the two variables, an individual's affordability and willingness, are taken into account. For example, if the two variables are positively correlated, the firm can increase quantity demanded by trying to increase the degree of correlation. However, an increase in quantity demanded does not mean an increase in the firm's revenue. Property 3 and 5 clearly state that when the price elasticity of demand is greater than one the correlation between consumer's affordability and willingness is positive, both quantity demanded and firm's sales revenue increase as correlation is increasing. If these two conditions can not be met, quantity demanded and sales revenue may work in opposite directions. For example, if price elasticity of demand is less than one, and consumers' income and their willingness are positively correlated, according to property 3 and 7, it is clear that quantity demanded will increase and the firm's sales revenue will decline as the correlation is increasing. Under the circumstance, the firm should ignore the marketing share but make every effort to raise revenue by implementing proper strategies to decrease the correlation. On the other hand, if consumers' income and their willingness are negatively correlated and the price elasticity of demand is greater than one, the firm can increase quantity demanded and sales revenue by lowering the degree of correlation  $(|\rho|)$  (i.e.  $\rho$  is increasing).

#### 6. Conclusions

This study is using bivariate normal distribution model to construct a market demand curve, and which still meet the law of demand. Besides, quantity demanded will increase when the consumers' income increase, and this is the basic characteristic of normal goods.

However, traditional approach didn't study the changes of quantity demanded and firm's revenue which are caused by the correlation between two variables, a consumer's affordability and willingness. The two variables are either positively or negatively correlated. Quantity demanded will change when the degree of correlation is changing. Based on the study, we find one of the interesting natures of inferior goods for the entire market. The nature is that consumers' affordability and willingness of purchasing an inferior good is negatively correlated, and even though the increase of the consumers' average income promotes their consumption, the "perverse" correlation effect will be strong enough to make total quantity demanded of an inferior good fall.

Now, we present the influence of correlation between income and willingness on both quantity demanded and the firm's sale revenue as Table 4.

Finally, following are the summary of major contributions of this study:

- (1) Firms can use market survey methods to find the estimated variables which construct a market demand curve for a specific good without using abstract utility function.
- (2) Firms can formulate their marketing strategies when the estimated correlation between the two variables, consumer's affordability and willingness, is taken into account. As the degree of correlation between these two variables changes, quantity demanded and the firms' revenue will also change.

Table 4

Changes of quantity demanded and the firm's revenue caused by the consumers' affordability and willingness.

Sign of correlation coefficient p	Price elasticity of demand  E <sub>d</sub>	Changes of correlation  p	Changes of quantity demanded	Changes of firm's revenue
Postive correlation $(\rho > 0)$	Price elasticity of demand is greater than 1	increases (increases of ρ)	increases	increases
		decreases (decreases of ρ)	decreases	decreases
	Price elasticity of demand is less than 1	increases (increases of ρ)	increases	decreases
		decreases (decreases of ρ)	decreases	increases
Negative Correlation (ρ < 0)	Price elasticity of demand is greater than 1	increases (decreases of ρ)	decreases	decreases
		decreases (increases of ρ)	increases	increases
	Price elasticity of demand is less than 1	increases (decreases of ρ)	decreases	increases
		decreases (increases of ρ)	increases	decreases

Based on this study, further studies can be conducted to explore the relationship between income inequality (i.e  $\sigma_x$  increase) and quantity demanded of a good. A further study can be conducted to explore the information spreading effect derived from actual trade prices in the market on the demand curve in the next period. The actual price information spreading in the consumers may let them reduce the degree of their differences among their pervious willingness (i.e.  $\sigma_y$  decrease). Then one can use this finding to construct the dynamic demand curve.

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